LAKELAND COMMUNITY COLLEGE – COURSE OUTLINE FORM*

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COURSE ID: **MATH2700**
COURSE TITLE: Calculus and Analytical Geometry III

	LECTURE	LAB	CLINICAL	TOTAL	OBR MIN	OBR MAX
CREDITS:	5.00	0.00	0.00	5.00	5.00	5.00
CONTACT HOURS:	5.00	0.00	0.00	5.00		

PREREQUISITE:

MATH 2500, MATH 2600; or permission of instructor

COURSE DESCRIPTION:

This is the third course in a three-semester sequence study of differential and integral calculus. Topics include vectors, differential calculus of functions of more than one variable, directional derivative, gradients, applications of partial derivatives, multiple integration, and line integrals. Students will need to supply a graphing utility; the instructor will provide details.

RATIONALE FOR COURSE:

This is the third course in a three-semester sequence study of differential and integral calculus.

OUTCOMES:
The course will

1. Present the fundamental concepts and basic techniques of multivariable calculus in a clear and concise manner and at a level suitable for sophomore engineering, mathematics, and science students.

2. Further develop students' ability to apply mathematical abstraction to concrete applications.

3. Develop students' understanding of and ability to use multivariable calculus as a tool but also how and why it works.

4. Develop students' ability to use theorems and definitions in combination.

5. Further develop mathematical abstraction, logical reasoning, the precision of a mathematical argument, and the construction of proofs.

6. Further develop the use of technology as a tool for determining solutions to real-life applications.

PERFORMANCE INDICATORS:
Upon completion of the course, the student should be able to

1. Add and subtract vectors and interpret the resultant geometrically.

2. Multiply a vector by a scalar and compute the dot product of two vectors and interpret these results geometrically.

3. Find the projection (orthogonal) of one vector onto another.

4. Take the cross products of two vectors and interpret the result geometrically.

5. Find the symmetric and parametric equations of lines in space.

6. Find the equation of a plane in space.

7. Calculate the area of a parallelogram and the volume of a parallelepiped in space.

8. Use scalar projections to determine the distance between a point and a line, between two lines, between a point and a plane, between a line and a plane, and between two planes.

9. Evaluate limits of vector-valued functions.

10. Determine intervals on which a vector-valued function is continuous.

11. Differentiate and integrate vector-valued functions.

12. Find the velocity and acceleration vectors at a point along a curve.

13. Find a position function by integration.

14. Find the unit tangent vector and the unit normal vector at a point along a curve.

15. Find the tangent line at a point on a curve in space.

16. Describe the motion of a projectile.

17. Find the arc length and curvature of a curve.

18. Find the tangential and normal components of acceleration.

19. Find the domain of a function of several variables.

20. Describe graphs, level curves, and level surfaces of functions of several variables.

21. Describe the graphs of quadric surfaces and cylinders.

22. Evaluate limits of functions of several variables.

23. Determine the continuity of a function of several variables.

24. Find and evaluate partial derivatives.

25. Find the slope of the tangent line to the curve of intersection of a surface in space and a plane that is perpendicular to the XY-plane and parallel to either the XZ-plane or the YZ-plane at a point on the curve.

26. Use partial derivatives to find rates of change.

27. Find higher-order partial derivatives.

28. Find the total differential.

29. Prove a function of two variables is differentiable at a given point using the definition of differentiability as well as applicable theorems.

30. Use a differential as an approximation.

31. Find derivatives and partial derivatives using the chain rules for functions of several variables.

32. Find derivatives and partial derivatives implicitly.

33. Find the gradient and directional derivative of a function.

34. Find the direction of maximum increase of a function at a point.

35. Find an equation of the tangent plane and an equation of a normal line to a surface at a given point on the surface.

36. Find the critical points of a function of several variables.

37. Determine the relative extrema and saddle points of a function of two variables using the second partials test.

38. Find the absolute extrema of a function of two variables over a closed region in the plane.

39. Find the absolute extrema of a function of two variables over open or unbounded regions, where possible.

40. Use Lagrange Multipliers to optimize subject to a constraint.

41. Solve applied optimization problems.

42. Evaluate iterated integrals.

43. Use an iterated integral to find the area of a region.

44. Switch the order of integration of an iterated integral.

45. Evaluate double integrals and use double integrals to find volume.

46. Change variables to polar coordinates, evaluate a double polar integral, and find areas of Polar Regions.

47. Use double integrals to find the center of mass, moments of inertia, and radius of gyration of a non-homogeneous region.

48. Use double integrals to find the area of a surface in space.

49. Evaluate triple iterated integrals and use triple integrals to find volume.

50. Use triple integrals to find the center of mass and the moments of inertia for a non-homogeneous solid region.

51. Plot points and graph basic equations in the cylindrical and spherical coordinate systems.

52. Convert points and equations between the rectangular, cylindrical, and spherical coordinate systems.

53. Evaluate and use triple integrals in cylindrical and spherical coordinates.

54. Find the gradient vector field for a given scalar function.

55. Determine whether or not a given vector field is conservative.

56. Identify inverse square fields.

57. Find a potential function for a given vector field.

58. Find the curl and divergence of a given vector field at a point and give a physical interpretation to these values.

59. Evaluate a line integral over a given path and use a line integral to find the work done by a given force field on a particle as it moves from one point to another along a curve in space.

60. Evaluate line integrals using the Fundamental Theorem of Line Integrals.

61. Determine whether or not a given line integral is independent of path.

62. Find work in a conservative force field.

63. Use Green's Theorem to evaluate a given line integral and to calculate work.

64. Find area by a line integral.

65. Find normal vectors to a surface defined parametrically.

66. Find the area of a surface defined parametrically.

67. Evaluate surface integrals.

68. Use surface integrals to find the flux of a field through surfaces defined explicitly and parametrically.

69. Verify and demonstrate The Divergence (Gauss's) Theorem by comparing the associated flux and triple integrals.

70. Verify and demonstrate Stokes's Theorem by comparing the associated work and flux integrals.

COURSE OUTLINE:

I. Vectors and the Geometry of Space
   - A. Vectors in the plane
   - B. Space coordinates and vectors in space
   - C. The dot product of two vectors
     1. The angle between vectors
     2. Orthogonality
     3. Projections
   - D. The cross product of two vectors in space
     1. Cross product and area
     2. The triple scalar product and volumes
   - E. Lines in space
     1. Parametric and symmetric forms
     2. Distance from a point to a line
   - F. Planes in space
     1. Traces in the coordinate planes
     2. Normal vectors
     3. Distance from a point to a plane
   - G. Vector-valued functions
   - H. Differentiation and integration of vector-valued functions
   - I. Velocity and acceleration
   - J. Tangent vectors and normal vectors
   - K. Arc length and curvature

II. Functions of Several Variables
- A. Introduction to functions of several Variables
  - 1. Domain
  - 2. Level curves and contour maps
    - a. cylinders and quadric surfaces
- B. Limits and continuity
  - 1. Neighborhoods in the plane and in space
  - 2. Limits along paths
  - 3. The precise definition of limit
- C. Partial derivatives
  - 1. Definition
  - 2. Geometric interpretation
  - 3. Interpretation as rates of change
- D. Differentials
  - 1. The total differential
  - 2. The definition of differentiability
    - a. relation to first partial derivatives
  - 3. Approximation
- E. Chain Rules for Functions of Several Variables
  - 1. Implicit differentiation
- F. Directional derivatives and gradients
  - 1. Definition of directional derivatives
    - a. relation to first partial derivatives
  - 2. Definition and properties of the gradient
    - a. relation to directional derivatives
    - b. direction of maximum increase/decrease
    - c. orthogonality to level curves and surfaces; normal vectors
- G. Tangent planes and normal lines
- H. Extrema of functions of two variables
  - 1. Critical points
  - 2. The Second Partials Test
    - a. relative extrema
    - b. saddle points
  - 3. The Extreme Value Theorem
    - a. absolute extrema on a closed, bounded region

- I. Applications of Extrema of Functions of Two Variables
  - 1. Optimization
- J. Lagrange multipliers

III. Multiple Integrations
- A. Iterated integrals and area in the plane
- B. Double integrals and volume
- C. Change of variables: polar coordinates
- D. Center of mass and moments of inertia
- E. Surface area
- F. Triple integrals and applications
- G. Triple integrals in cylindrical and spherical coordinates
  - 1. The cylindrical and spherical coordinate systems
- H. Change of variables: Jacobians (as time permits)

IV. Vector Analysis
- A. Vector fields
  - 1. Divergence and curl
  - 2. Potential functions
  - 3. Conservative fields
  - 4. Inverse square fields
- B. Line integrals
  - 1. Work done moving a particle along a path in a force field
- C. Conservative vector fields and independence of path
  - 1. The Fundamental Theorem of Line Integrals
- D. Green's Theorem
  - 1. Computing an area from a line integral

2. Flux – divergence form
3. Circulation – curl form

E. Parametric surfaces
1. Normal vectors
2. Surface area

F. Surface integrals
1. Flux

G. Divergence Theorem

H. Stokes's Theorem

INSTRUCTIONAL PROCEDURES THAT MAY BE UTILIZED:

Lecture/discussion
Computer/graphing calculator based activities
Group and/or individual activities
Research projects utilizing real data gathered from the Internet or other sources

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GRADING PROCEDURES:

It is recommended that the instructors have at least five evaluative items on which to determine the student's course grade. In general, tests are given covering lecture and homework assignments.

COURSE EVALUATION PROCEDURES:

Student course evaluations
Student success rate in subsequent mathematics courses

See pages 17-19 of Curriculum Procedures & Guidelines for definitions of course outline terms.

LAKELAND LEARNING OUTCOMES

		I	R	D
	LEARNS ACTIVELY			
1.	Takes responsibility for his/her own learning.			D
2.	Uses effective learning strategies.			
3.	Reflects on effectiveness of his/her own learning strategies.			
	THINKS CRITICALLY	**I**	**R**	**D**
4.	Identifies an issue or idea.			
5.	Explores perspectives relevant to an issue or idea.			
6a.	Identifies options or positions.			
6b.	Critiques options or positions.			
7.	Selects an option or position.			D
8a.	Implements a selected option or position.			
8b.	Reflects on a selected option or position.			
	COMMUNICATES CLEARLY	**I**	**R**	**D**
9a.	Uses correct spoken English.			
9b.	Uses correct written English.			
10.	Conveys a clear purpose.			
11.	Presents ideas logically.			D
12a.	Comprehends the appropriate form(s) of expression.			D
12b.	Uses the appropriate form(s) of expression.			D
13.	Engages in an exchange of ideas.			
	USES INFORMATION EFFECTIVELY	**I**	**R**	**D**
14.	Develops an effective search strategy.			
15a.	Uses technology to access information.			D
15b.	Uses technology to manage information.			
16.	Uses selection criteria to choose appropriate information.			D
17.	Uses information responsibly.			
	INTERACTS IN DIVERSE ENVIRONMENTS	**I**	**R**	**D**
18a.	Demonstrates knowledge of diverse ideas.			
18b.	Demonstrates knowledge of diverse values.			
19.	Describes ways in which issues are embedded in relevant contexts.			
20a.	Collaborates with others.			
20b.	Collaborates with others in a variety of situations.			
21.	Acts with respect for others.			

Definitions:

Introduces (I)

Students first learn about key ideas, concepts, or skills related to the performance indicator. This usually happens at a general or very basic level, such as learning one idea or concept related to the broader outcome.

Reinforces (R)

Students are given the opportunity to synthesize key ideas of skills related to the performance indicator at increasingly proficient levels.

Demonstrates (D)

Students should demonstrate mastery of the performance indicator with the level of independence expected of a student attaining an associate’s degree.