LAKELAND COMMUNITY COLLEGE – COURSE OUTLINE FORM

ORIGINATION DATE:	8/2/99	**APPROVAL DATE:**	2/27/23
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COURSE ID: **MATH2800**

COURSE TITLE: Linear Algebra

	LECTURE	LAB	CLINICAL	TOTAL	OBR MIN	OBR MAX
CREDITS:	4.00	0.00	0.00	4.00	4.00	4.00
CONTACT HOURS:	4.00	0.00	0.00	4.00		

PREREQUISITE:

MATH 2500, MATH 2600; or permission of instructor

COURSE DESCRIPTION:

This course includes a study of systems of linear equations, matrix algebra, determinants, vector spaces, linear transformations, eigenvalues, eigenvectors, diagonalization, and applications. Students will need to supply a graphing utility; the instructor will provide details.

RATIONALE FOR COURSE:

This course includes a study of systems of linear equations, matrix algebra, determinants, vector spaces, linear transformations, eigenvalues, eigenvectors, diagonalization, and applications. It is designed for students planning to transfer to a mathematics, physics, engineering, or computer science four-year program.

OUTCOMES:
The course will

1. Present the fundamental concepts and basic techniques of linear algebra in a clear and concise manner and at a level suitable for sophomore engineering, mathematics, and science students.

2. Provide a further study of mathematical abstraction, logical reasoning, the precision of a mathematical argument, and the construction of proofs.

3. Further develop students' ability to apply mathematical abstraction to concrete applications.

4. Develop students' understanding of and ability to use linear algebra as a tool.

5. Continue to develop students' ability to use theorems and definitions in combination.

6. Further develop the use of technology as a tool for determining solutions to real-life applications.

PERFORMANCE INDICATORS:
Upon completion of the course, the student should be able to

1. Solve systems of equations using matrices and determinants.
2. Add, subtract, and multiply matrices.
3. Multiply a matrix by a scalar and determine the transpose of a given matrix.
4. Find the inverse of a matrix.
5. Evaluate the determinant of a matrix and interpret geometrically.
6. Describe the affect of row operations on the value of a determinant.
7. Find the adjoint of a matrix.
8. Demonstrate R^n satisfies the axioms for a vector space.
9. Determine if specified subsets of R^n are subspaces of R^n.
10. Show that a specified set of vectors in R^n is linearly independent.
11. Show that a given set of vectors forms a basis for a specified subspace of R^n.
12. Exhibit a basis, and calculate the dimension of a given subspace of R^n.
13. Find a basis for a given subspace of R^n that includes a given vector.
14. Find a basis for the row and column space of a matrix, and determine the rank of a matrix.
15. Find the coordinates of a vector with respect to a specified basis of R^n.
16. Demonstrate properties of the dot product of vectors in R^n.
17. Obtain an orthogonal basis for a subspace of R^n by normalizing a given set of vectors.
18. Show that a set of vectors in R^n is orthogonal.
19. Show that a set of vectors is an orthogonal basis for a given subspace of R^n.
20. Normalize the rows of a matrix to make it orthogonal.
21. Use the Gram-Schmidt algorithm to convert a given basis of a subspace of R^n into an orthogonal basis.
22. Find the characteristic polynomial, eigenvalues, and the eigenvectors of a given matrix.
23. Determine the algebraic and geometric multiplicities of an eigenvalue.
24. Show that two matrices are not similar by computing the trace, determinant, and rank.
25. Determine if a matrix is diagonalizable.
26. Diagonalize a matrix using a basis of eigenvectors.
27. Show that a function between R^n and R^m is a linear transformation.
28. Given a matrix, find the range and nullspace; rank and nullity.

29. Given a matrix, find a basis for the kernel and image of the corresponding matrix transformation; and find the rank and nullity of the transformation.

30. Determine whether a given transformation has an inverse.

31. Prove elementary facts about vector and matrix operations using the appropriate definitions and notation.

32. Prove elementary facts about the nullspace and range, nullity and rank of a matrix.

33. Define vector space and determine if given sets under specific operations are vector spaces over specified fields.

34. Use vector space axioms and associated definitions to prove results about vector spaces such as the uniqueness of inverses.

COURSE OUTLINE:

I. Vectors in R^n
  A. Algebraic and geometric representations
  B. Operations with vectors
    1. Addition and its Properties
      a. Parallelogram Law
      b. proofs of the properties
    2. scalar multiplication and its properties
      a. proofs of the properties
    3. the dot product and its properties
      a. the angle between vectors
      b. orthogonality
      c. proofs of the properties

II. Systems of Linear Equations
  A. Introduction to systems of linear equations
  B. Gauss-Jordan elimination
    1. equivalent systems of equations
    2. Row-Echelon matrices
  C. Solutions of systems of equations
    1. geometric interpretations
    2. theorems about solutions of systems
      a. homogeneous systems
      b. non-homogeneous systems

III. Matrix Algebra
  A. Matrix addition and its properties
    1. proofs of the properties.
  B. Scalar multiplication and its properties
    1. proofs of the properties.
  C. Matrix multiplication and its properties
    1. non-commutativity
    2. proofs of the properties.
  D. Matrix inverses
    1. definition and basic properties
      a. proofs of the basic properties.
    2. solutions of systems of linear equations using matrix inverses
    3. elementary matrices
  E. The transpose of a matrix and its properties
    1. proofs of transpose properties

IV. Determinants
  A. The Laplace expansion
    1. cofactor expansion
    2. evaluating determinants by row reduction

- B. Determinants and matrix inverses
    1. properties of the determinant and their proofs.
    2. Cramer's Rule
- C. Geometric properties of the determinant

V. R^n as a Vector Space
- A. Basic properties
- B. Subspaces
    1. definition and properties
    2. subspace criteria theorems and their proofs
    3. the span of a set of vectors
        a. spanning sets of vectors
- C. Linear independence and dimension
    1. linear independence
    2. basis and dimension
    3. existence of bases
- D. Coordinates and change of basis

VI. Eigenvalues and Diagonalization of Matrices
- A. Eigenvalues and eigenvectors
    1. eigenvalues and the characteristic polynomial
        a. algebraic multiplicity of eigenvalues
    2. basic theorems concerning eigenvectors and their proofs.
- B. Eigenspaces
    1. geometric multiplicity of eigenvalues
- C. Similarity and diagonalization
    1. similar matrices
    2. bases of eigenvectors

VII. Linear Transformations from R^n to R^m
- A. Definition and basic properties
    1. examples and elementary properties
    2. kernel and image of a linear transformation
        a. the Dimension Theorem
    3. composition and inverse
- B. Matrices and linear transformations
    1. the matrix of a linear transformation
        a. matrix multiplication and composition of transformations
        b. matrix inverses and inverses of transformations
    2. the row space and column space of a matrix
        a. rank and nullity
        b. the Dimension Theorem
    3. matrices and geometric transformations in the plane.
    4. the vector space of linear transformations

VII. Abstract Vector Spaces
- A. Vector space axioms and examples
    1. m by n matrices
    2. polynomials of degree n or less
- B. Subspaces (as time permits)
    1. theorems about subspaces
- C. Spanning sets, basis, and dimension in Abstract Vector Spaces (as time permits)
- D. Linear transformations between abstract vector spaces (as time permits)
    1. kernel and image
    2. the Dimension Theorem
    3. representation by matrices
    4. isomorphism
        a. coordinates and change of basis

VIII. R^n as an Inner Product Space (as time permits.)
- A. Vector norms in R^n

        1. Cauchy Schwartz Inequality and its proof
        2. Triangle Inequality and its proof
    B. Orthogonality
        1. orthogonal sets of vectors
        2. projections and the Gram-Schmidt algorithm
        3. application to least squares problem
    C. Orthogonal diagonalization of symmetric matrices

IX. Numerical Methods (as time permits)
    A. Solutions of systems of equations
    B. Determining eigenvalues and eigenvectors
    C. QR factorization

INSTRUCTIONAL PROCEDURES THAT MAY BE UTILIZED:

Lecture/discussion
Computer/graphing calculator based activities
Group and/or individual activities
Research projects utilizing real data gathered from the Internet or other sources

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GRADING PROCEDURES:

It is recommended that the instructors have at least five evaluative items on which to determine the student's course grade. In general, tests are given covering lecture and homework assignments.

COURSE EVALUATION PROCEDURES:

Student course evaluations
Student success rate in subsequent mathematics courses

LAKELAND LEARNING OUTCOMES

		I	R	D
	LEARNS ACTIVELY			
1.	Takes responsibility for his/her own learning.			D
2.	Uses effective learning strategies.			
3.	Reflects on effectiveness of his/her own learning strategies.			
	THINKS CRITICALLY	**I**	**R**	**D**
4.	Identifies an issue or idea.			
5.	Explores perspectives relevant to an issue or idea.			
6a.	Identifies options or positions.			
6b.	Critiques options or positions.			
7.	Selects an option or position.			D
8a.	Implements a selected option or position.			
8b.	Reflects on a selected option or position.			
	COMMUNICATES CLEARLY	**I**	**R**	**D**
9a.	Uses correct spoken English.			
9b.	Uses correct written English.			
10.	Conveys a clear purpose.			
11.	Presents ideas logically.			D
12a.	Comprehends the appropriate form(s) of expression.			D
12b.	Uses the appropriate form(s) of expression.			D
13.	Engages in an exchange of ideas.			
	USES INFORMATION EFFECTIVELY	**I**	**R**	**D**
14.	Develops an effective search strategy.			
15a.	Uses technology to access information.			D
15b.	Uses technology to manage information.			
16.	Uses selection criteria to choose appropriate information.			D
17.	Uses information responsibly.			
	INTERACTS IN DIVERSE ENVIRONMENTS	**I**	**R**	**D**
18a.	Demonstrates knowledge of diverse ideas.			
18b.	Demonstrates knowledge of diverse values.			
19.	Describes ways in which issues are embedded in relevant contexts.			
20a.	Collaborates with others.			D
20b.	Collaborates with others in a variety of situations.			
21.	Acts with respect for others.			

Definitions:

Introduces (I)

Students first learn about key ideas, concepts, or skills related to the performance indicator. This usually happens at a general or very basic level, such as learning one idea or concept related to the broader outcome.

Reinforces (R)

Students are given the opportunity to synthesize key ideas of skills related to the performance indicator at increasingly proficient levels.

Demonstrates (D)

Students should demonstrate mastery of the performance indicator with the level of independence expected of a student attaining an associate’s degree.